1) The simple series circuit of the varactor can be used with good accuracy if the value of R_k is allowed to take on different values at different frequencies.

2) Pumping can be accurately represented as sinusoidal current when the varactor mount is a simple resonance waveguide cavity of the type used in the model amplifier.

3) If the cutoff frequency is measured by Kurokawa's [1] method over the bias sweep range of $2(V_0+\Phi)$, where V_0 is the working self-bias voltage of the amplifier operated at the required gain, the value obtained is the exact value achieved in the amplifier "as pumped." Other methods for cutoff frequency require extrapolation and are less accurate.

4) Using the assumption of 3), the value of m_1 is always the "fully pumped value."

From the circuit equation an intuitive "flow chart" for the operation of a parametric amplifier has been developed, Fig. 3.

Finally a design method has been presented which exposes the contributing factors of parametric amplifier performance individually, allows their independent measurement, and gives information for the accurate prediction of the amplifier performance.

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Phase Equalization Problems in a Phased-Array Transmitter

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Abstract—**Current radar systems employing programmed multiple antenna feeds to orient the antenna beam electronically require large numbers of parallel channels operating simultaneously with nearly identical phase-delay characteristics. These channels include active devices such as microwave amplifiers for which phase-delay, as well as gain and power output, must be identical to accomplish high pointing accuracy and maximum power addition. When the system is required to operate over a wide frequency band, all channels must track in phase.**

This paper describes problems encountered in the development of a prototype transmitter requiring multiple channel phase-delay equality. A new medium power pulsed TWT is described and measurements of its electrical length are discussed. Computer programming which reduced the vast amount of phase information derived from this development is described.

Phase Equalization

A BLOCK DIAGRAM of the transmitter under consideration is shown in Fig. 1. The maximum overall system phase error (about 35° rms) was established by the acceptable signal degradation (about 2 dB). A division of tolerances for the transmitter resulted in a 15° rms allowance for the power amplifier module.

Because the power amplifier modules must be interchangeable, it was required to equalize their electrical path lengths by referring their phase vs. frequency curves to an absolute reference and then compensating to obtain the best mean square fit to an average. Single frequency compensation may then be introduced to minimize wide-band phase tracking errors.

By maintaining close manufacturing tolerances on the several types of transmission line and on other passive devices (transitions, transformers, couplers, circulators, connectors, etc.) to minimize their VSWR, wide-band phase excursions are held to a minimum. Phase considerations for active devices, such as microwave power amplifier tubes, are more complex. Cumulative manufacturing tolerances for the many tube parts usually are not compatible with system phase requirements.

For instance, suppose we assume a standard or average phase characteristic as shown in Fig. 2. Assume another phase characteristic to be the curve for any tube, say the Kth tube. Let $\phi_{K,i}$ be the phase measured for this tube at frequency i. We wish to refer this curve to the standard such that the average value of its algebraic deviations, when overlayed on the standard within a specified frequency band, is zero. Expressed mathematically,

$$\phi'_{K,i} = \phi_{K,i} + \phi_{K,0} \tag{1}$$

where

$$\phi_{K,0} = \frac{1}{NJ} \sum_{i=1}^{J} \sum_{j=1}^{N} \phi_{j,i} - \frac{1}{J} \sum_{i=1}^{J} \phi_{K,i} \tag{2}$$

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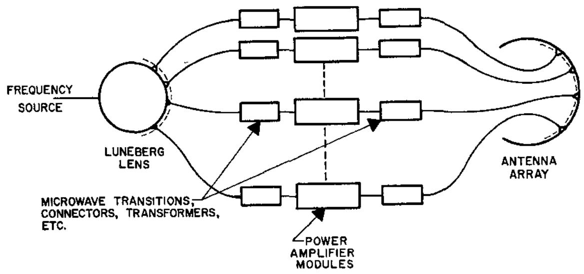

Fig. 1. Block diagram—phased array transmitter.

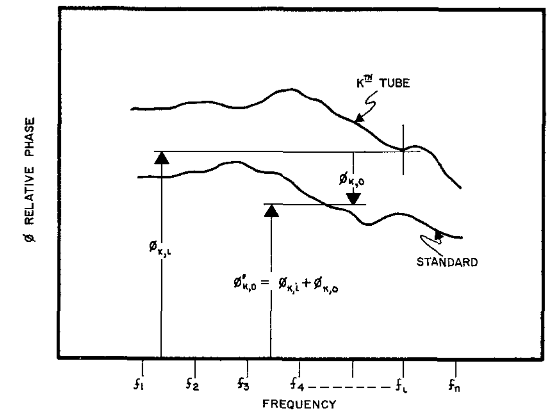

Fig. 2. Phase-frequency characteristic.

In words then, $\phi_{K,0}$ is equal to the average of all phases for all tubes minus the average phase for the Kth tube at all frequencies. $\Phi'_{K,0}$ is then the desired final phase setting for all tubes.

Equipment Description

The power amplifier module shown in Fig. 3 includes the traveling-wave tube (TWT) and associated input and output transverse electromagnetic (TEM) mode transmission line assemblies. The input assembly includes miniature coax cable, a phase shifter and a strip-line Y-circulator functioning as an isolator. The miniature coax cable has an outer conductor of seamless, extruded metal tubing and utilizes new miniature coax connectors developed by the Bendix Research Labs. and Omni-Spectra, Inc. The output assembly includes rigid air-line coaxial cable and a coaxial directional coupler.

The power amplifier tube developed for this program is a high-gain, grid controlled traveling-wave tube. Its phase-delay (electrical length) and phase-tracking characteristics were unknown. Initially, these are functions of the tube folded-waveguide slow-wave structure, the voltages applied to its control elements, its input and output impedance match and of system operating conditions. Specification limits required of these tubes, in order that parallel use of them satisfy radar transmitter requirements, are listed as follows:

$\Delta\phi_d$ (DUTY)	10° Nominal
$\Delta\phi_d$ (VSWR)	13° Maximum
$\Delta\phi_d$ (RF)	20° Nominal

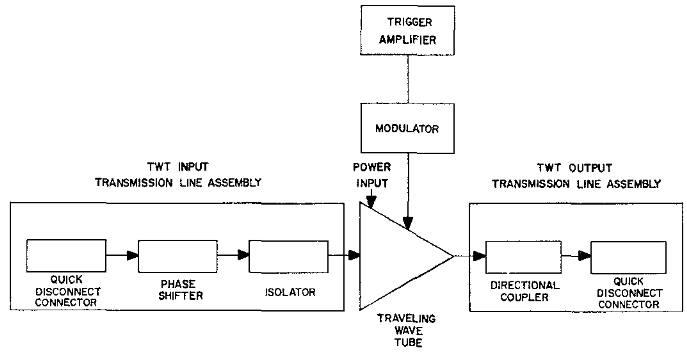

Fig. 3. Block diagram—power amplifier module.

$\Delta\phi_d$ (DUTY) is nominal phase change for change in duty cycle from 0.01 to 0.024,

$\Delta\phi_d$ (VSWR) is maximum phase change for change in phase of a 1.5:1 VSWR load of 360°, and

$\Delta\phi_d$ (RF) is nominal phase change for change in RF drive from the nominal value to 6 dB below the nominal value.

All limits are as previously specified unless,

$$26° \geq \sqrt{\overline{\Delta\phi_d(\text{DUTY})^2} + \overline{\Delta\phi_d(\text{VSWR})^2} + \overline{\Delta\phi_d(\text{RF})^2}}$$

where $\Delta\phi_d$(VSWR) does not exceed 13°.

The phase sensitivity factors for the tube are specified to be less than the values listed as follows:

$$\frac{\Delta\phi_d}{\Delta E_f} \leq +5°/\text{V}$$

$$\frac{\Delta\phi_d}{\Delta e_c} \leq +3°/\text{V}$$

$$\frac{\Delta\phi_d}{\Delta E_b} \leq -0.13°/\text{V}$$

$$\frac{\Delta\phi_d}{\Delta T} \leq 0.1°/°\text{F}$$

where

$\Delta\phi_d/\Delta E_f$ is phase change as a function of heater voltage,

$\Delta\phi_d/\Delta e_c$ is phase change as a function of grid to cathode pulse amplitude,

$\Delta\phi_d/\Delta E_b$ is phase change as a function of cathode-to-shell voltage and

$\Delta\phi_d/\Delta T$ is phase change as a function of temperature.

The average of phase delay deviations for each tube from standard values for the six test frequencies when operated under standard test conditions shall not exceed $\pm 7.5°$.

Specific Phase Problems

The tube requirement specification allows the phase delay to have any absolute value commensurate with the structure required to meet performance specifications. The variations in phase delay from tube to tube are, of course, required to have a very limited distribution. The practical approach to multiple-channel equalization is to select a standard tube or "norm," from a representative sample lot of tubes and reference the phase delay of all other tubes to this standard. In this particular development work, manufacturing facilities at three widely separated locations were employed, and the data compiled indicated that the tubes being produced by the two tube suppliers were actually different in electrical length. Values for relative phase were measured at discrete frequencies uniformly spaced across the band, and this data, when plotted, yielded different phase-frequency slopes resulting in an apparent double-standard. The resulting two "standard" curves are shown in Fig. 4, normalized at F_1 by 234°. It is obvious that the path lengths for each supplier's bridge were not well balanced and were unbalanced in different directions. Not knowing whether this difference was due entirely or partly to measuring techniques, there remained the possibility that the tubes of each supplier actually have different values for absolute phase delay. Of course, if this were so, even tubes having perfectly linear phase characteristics would not phase track with each other and consequently would not be interchangeable in the radar transmitter.

Because of these problems, it was decided to measure the values for absolute phase delay of the transmitter module and its associated microwave components, the TWT and its input and output transmission lines. For these tests, a coaxial cable, whose electrical length is approximately equal to that of the "live" module, was assembled into a module housing and is employed as a standard. The ϕ_d's of the transmission lines and the standard were measured by the conventional microwave short-circuit technique and are listed for six uniformly spaced frequencies:

Frequency	ϕ_d Input line	ϕ_d Output line	ϕ_d Standard
F_1	8465	2275	23,211
F_2	8615	2308	23,645
F_3	8770	2343	24,076
F_4	8925	2396	24,510
F_5	9085	2440	24,946
F_6	9240	2472	25,366

We knew of no accurate way of making a direct measurement of the absolute phase delay of a 40-dB gain device, such as the TWT. Therefore, it was decided to use the substitution method in a microwave bridge [2], which permitted a comparison of the ϕ_d's of the "live" power amplifier (PA) module and the standard. In this comparison, the TWT ϕ_d (absolute) at any frequency must be deduced from the bandwidth phase slope. An analysis of the results shows that although the ϕ_d of the standard is accurately known, the values for the TWT and PA module ϕ_d's have a possible error of an integral number of wavelengths. The most prominent reason for this is the nonlinearity of the TWT phase-frequency slope.

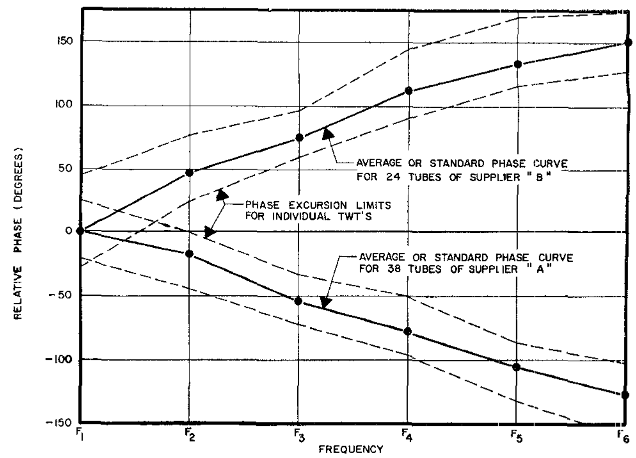

Fig. 4. Phase-frequency characteristics for TWT's of two suppliers.

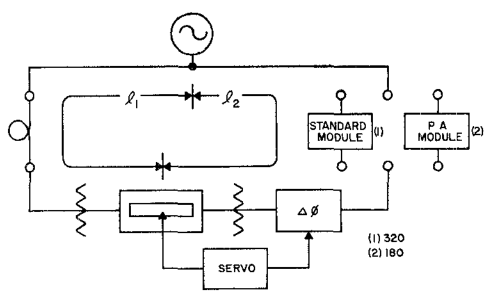

Fig. 5. Microwave bridge.

Consider the functional block diagram of the microwave bridge, shown in Fig. 5.

A servo loop assures a fixed position of the null of the slotted line by driving the phase shifter. Consequently, the phase shifter readings are direct indications of phase differences.

With the standard module installed in the microwave bridge, the reading on the bridge phase shifter at F_1 is 320. All the PA modules were adjusted for a reading on the bridge phase shifter at F_1 of 180. The slope of the PA module characteristic increases with frequency, and this tells us that the PA module must be shorter than the dummy module whose characteristic is flat except for a slight (7°) bow (which indicates only the difference in dispersion between the two bridge paths). This is illustrated in Fig. 6.

The phase shifter reading is ambiguous because the RF signal repeats itself every 360°, the reading can be 180 plus an integral number n of wavelengths, i.e., $180+n\lambda$. For instance, it could be $180+360=540°$, or $180+2(360)=900°$, where $n=1$ and 2, respectively. In this particular case, where we can approximate the slope of the PA module phase-frequency curves, the value of n appears to be limited to about ± 2.

The slope is equal to the phase difference at F_0 modified by the percentage bandwidth. If 540° were the correct number, the slope would be $(540-320)$ by $(f_6-f_1)/f_1$ $=220$ by 9.3 per cent $=20°$. If 900 were the correct number, the slope would be 54°. The high frequency readings for these two slopes would then be 200° and 234°, respectively.

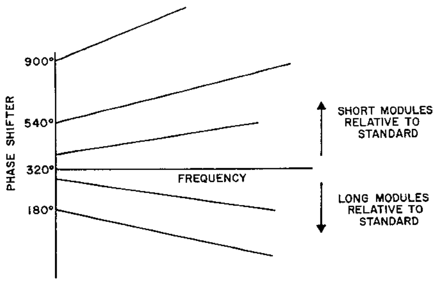

Fig. 6. Module relative phase characteristic.

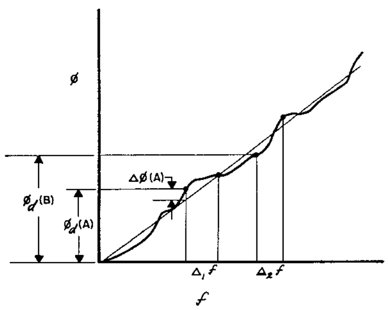

Fig. 7. Phase-frequency characteristic.

When these slopes are overlaid on several curves of the PA module ϕ-f characteristic, one of the slopes seems to be the average for some curves while the other slope seems to "fit" other curves. If the module slope were linear there would be no question about which slope were the correct one and the module ϕ_d and TWT ϕ_d could be calculated accurately at every frequency. However, the nonlinearity in the slope of the TWT is greater than the phase change for a bridge path length difference of one wavelength, so that it is impossible to be sure (by this technique alone) whether the slope is due to TWT variations or bridge path length difference.

We can look at this ambiguity from a more general point of view. Consider the ϕ-f characteristic of a filter. Assume the characteristic shown in Fig. 7 taken from zero frequency.

It is not feasible to calculate ϕ_d for a point on the curve from the slope for a narrow frequency range. For instance, $\phi_d(A)$ cannot be deduced from the slope of the limited frequency range, $\Delta_1 f$. If the characteristic could be plotted to zero, the average slope (a straight line) might be determined and the deviation from this straight line, such as $\Delta\phi(A)$, could be determined for any point. Since our TWT is an active (rather than passive) device, designed to operate over only a narrow frequency band, its ϕ-f characteristic cannot be plotted to zero frequency. Another consideration is that the average slope of the TWT is not linear because its slow-wave structure is a form of waveguide. Its phase-frequency characteristic must include some dispersion. The problem continues to be one of not knowing the exact slope for the frequency range under consideration. Since the true TWT phase slope is not known, there is no way of being certain (with this technique alone) of the absolute phase delay of the TWT, or of the PA module.

Results

One way to specify the TWT phase-frequency characteristic is to cancel out integral numbers of wavelengths and simply specify the relative phase, i.e., the phase at the tube output relative to its input. If we do this, we arrive at the following values:

Frequency	Probable ϕ_d-PA module	ϕ_d *XMSN* lines	Probable ϕ_d TWT	ϕ_d (Relative) TWT
F_1	22,991°	10,740°	12,251°	11°
F_2	23,435°	10,923°	12,512°	272°
F_3	23,856°	11,113°	12,743°	143°
F_4	24,285°	11,321°	12,964°	4°
F_5	24,708°	11,525°	13,183°	223°
F_6	25,101°	11,712°	13,389°	69°

Possible errors and uncertainty included in the determination of TWT ϕ_d are:

	Uncertainty	Error
Microwave bridge ambiguity ($n\lambda$)*	nx 360°	
Input line measurement		±10°
Output line measurement		±15°
Standard module measurement		±10°
Bridge comparison of modules		+3°
Sample lot error		±20°
Total possible error	—	±58°

* n is any integer.

Transmitter modules are adjusted for microwave phase delay during production testing. Although an absolute reference and the slope of the phase characteristic are functions of a particular measuring system, the deviation at any frequency from tube-to-tube is *not* dependent upon the tracking characteristic of the measuring system. And so, if the measurement accuracy is known to be good, an accurate module-to-module comparison can be made. When the swept-frequency phase curves for a small quantity of modules had been run, a plot of the average for these curves was used as the phase tracking "standard." Figure 8 shows the standard and typical tube curves.

When the testing of approximately 200 modules (400 TWT's) had been completed, it was found that the phase excursion for only twelve (or 3 per cent) of the TWT's (and associated input and output assemblies) exceeded ±30° peak. This deviation beyond the spec value was in all cases limited to a small portion, usually less than 5 per cent, of the frequency band.

Because of the nature of multiple channel addition of power for the formation of a transmitter beam, it is appropriate to redefine allowable phase-tracking excursion for the PA module as an rms deviation. If we allow an average deviation of ±7.5°, then assuming a sinusoidal function, this would have an rms value of 8.4°. If we allow a 4° peak phase deviation for the TWT input line and 4° for the TWT output line, this allows 2.8° rms for each line. Thus, the total error allowable for a PA module is

$$\sqrt{\overline{8.4^2} + \overline{2.8^2} + \overline{2.8^2}} = 9.3°$$

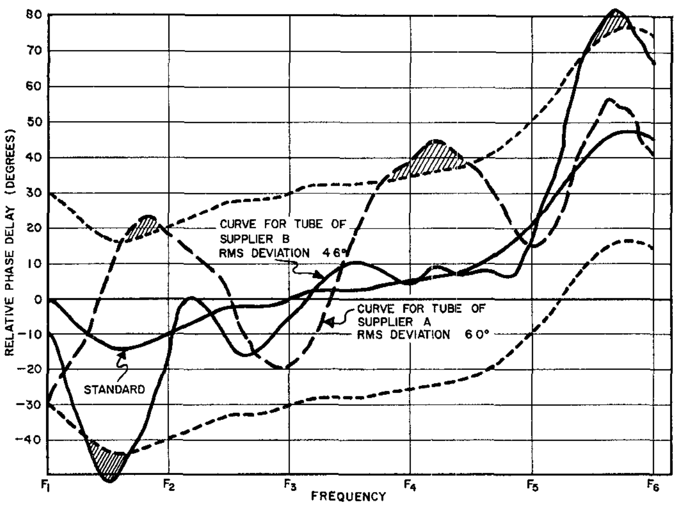

Fig. 8. Phase characteristics for two TWT's for which deviation exceeds $\pm 30°$.

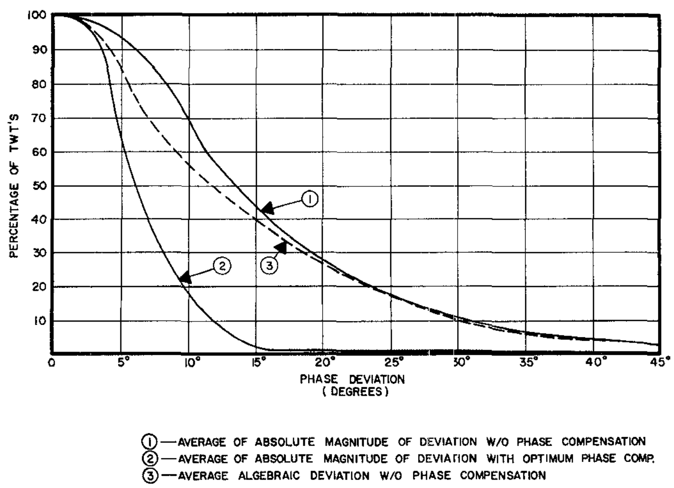

Fig. 9. Percentage of TWT's with deviation equal to or greater than degrees indicated.

The average of the rms values for a sample of good curves was 4° rms. Only one of the twelve TWT assemblies whose phase exceeded $\pm 30°$ peak, failed to pass the 9.3° rms specification. Thus, all but 0.3 per cent of the TWT assemblies met the phase-tracking specification after their delays were optimized with phase shifter adjustment.

Computer Program

The accumulation of a vast amount of phase data suggested the use of computer programming to process the data, and so an IBM 7094 was employed to determine (for all available tube data—approximately 1000 tubes), at six frequencies:

1) The true standard phase curve
2) The algebraic deviations of every tube from the standard
3) The average of the absolute magnitude of deviations (neglecting algebraic sign) prior to any phase adjustment external to the tube
4) The average of the algebraic sum of deviations
5) The average of the numeric deviations as in 3), after optimum compensation, from operation 4), is introduced.

The benefits of this program are obvious. The first operation revealed that our standard curve was in error by 15°. Results of the second and third operations indicate the phase quality of the TWT independent of application. The negative value of the average yielded in the fourth operation is the amount of single-frequency phase compensation required for each tube to obtain optimum overlay of its phase characteristic on the standard. Thus, it is the amount of compensation necessary to make the tube useable in a multichannel, phase coherent system. This mathematical correction assumes that there is no limit to the range of adjustment. This, of course, is not the case in practice, and is cause for the rejection of some modules during the test phase of production. Results of the fifth operation show how well the tubes track with each other after single-frequency compensation has been included; it also indicates the contribution of each tube to transmitter system power losses. The results of operations three and five are shown in Fig. 9.

Conclusions

1) To our knowledge, it is the first time that data has been compiled in such large quantities as to show meaningful distribution for this type of tube. Of course, the TWT discussed herein was developed specifically for this prototype transmitter application, and so it is the first record of such data for this particular tube. It should be selected for use in a phase-coherent system only if phase compensation external to the tube is provided which makes it possible to reduce the tube-to-tube spread of phase deviation to an acceptable level.
2) Computer programming has proved to be of invaluable assistance in expediting the reduction of vast amounts of tube data, making it possible to keep an updated record of the standard to which new tube modules are optimized. In addition, the highly successful performance of the automatic phase measuring system provided quick access to a large quantity of information on which improvements in the equipment were based.
3) It would be desirable, from the system designer's point of view, to specify the TWT phase delay in absolute terms. However, the measurement of absolute values appear to be very difficult. Fortunately, as this program has verified, relative measurements, when properly made, are satisfactory.

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